APPENDIX B – GEOMETRIC AND TRIGONOMETRIC REFERENCE CHARTS

The charts in this section contain a number of formulas for performing geometric and trigonometric calculations. These charts should be used as references in calculating quantities for measurement and payment.

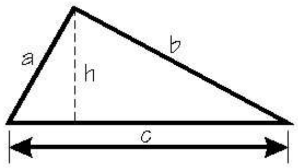
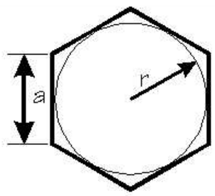
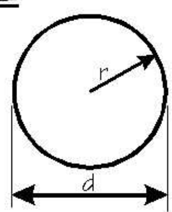
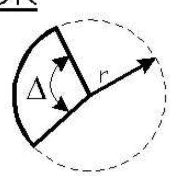
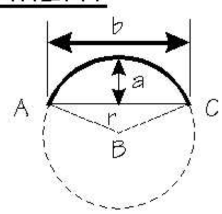
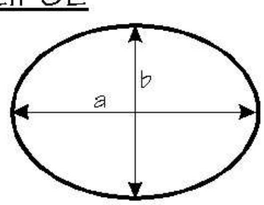
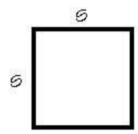
OBLIQUE TRIANGLE  h = altitude c = base $s = \dfrac{a+b+c}{2}$	$A = \dfrac{ch}{2}$ or, $A = \sqrt{s(s-a)(s-b)(s-c)}$
REGULAR POLYGON  Example: Polygon (hexagon) with circle inscribed within.	$A = \dfrac{\text{Sum of sides X r}}{2}$
CIRCLE  r = radius $r = \dfrac{\text{circum}}{2\pi}$ d = diameter $d = \dfrac{\text{circum}}{\pi}$ $\pi = 3.1416$ circum $= \pi d$	$A = \pi r^2$ $A = .7854\ d^2$ $A = \dfrac{\text{circum X r}}{2}$ $A = \dfrac{\pi d^2}{4}$
SECTOR  r = radius Δ = Degrees of Enclosed Angle	$A = \dfrac{\text{Area of Circle X } \Delta}{360^\circ}$
SEGMENT 	A = 2/3 ab (Approximately) Use for segments having M.O. of less than 1/4 the length of the chord A=0.7854ab Use for segments
ELLIPSE  a = Major Axis b = Minor Axis	A = .7854 ab

Figure J-1: Areas of Common Geometric Shapes

SQUARE

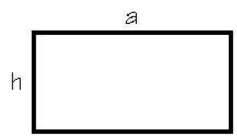


All Angles = 90°

$s = s$

$A = s^2$

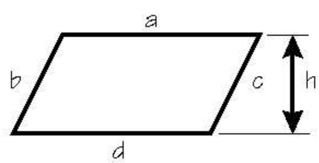
RECTANGLE



All Angles = 90°

$A = ah$

PARALLELOGRAM



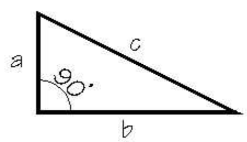
$a = d$

$b = c$

Opposite Sides are Parallel

$A = ah$

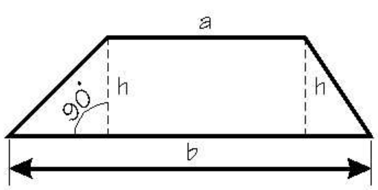
RIGHT TRIANGLE



$a^2 + b^2 = c^2$

$$A = \frac{ab}{2}$$

TRAPEZOID



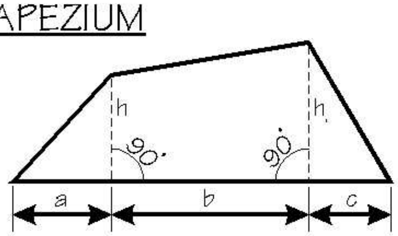
a parallel to b

$$A = \frac{(a + b)}{2} h$$

or

Solve as Sum of Rectangle and 2 Triangles

TRAPEZIUM

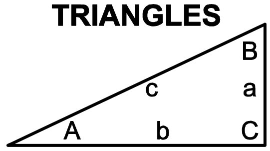


h parallel to h_1

$$A = \frac{b(h + h_1) + ah + ch_1}{2}$$

Figure J-2: Areas of Common Geometric Shapes

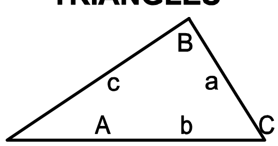
RIGHT ANGLED TRIANGLES



$$a^2 = c^2 - b^2$$
$$b^2 = c^2 - a^2$$
$$c^2 = a^2 + b^2$$

Known	Required					
	A	B	a	b	c	Area
a, b	$\tan A = \frac{a}{b}$	$\tan B = \frac{b}{a}$			$\sqrt{a^2 + b^2}$	$\frac{ab}{2}$
a, c	$\sin A = \frac{a}{c}$	$\cos B = \frac{a}{c}$		$\sqrt{c^2 - a^2}$		$\frac{a\sqrt{a^2 + b^2}}{2}$
A, a		$90^0 - A$			$\frac{a}{\sin A}$	$\frac{a^2}{2 \tan A}$
A, b		$90^0 - A$	$b \tan A$		$\frac{b}{\cos A}$	$\frac{b^2 \tan A}{2}$
A, c		$90^0 - A$	$c \sin A$	$c \cos A$		$\frac{c^2 \sin 2A}{4}$

OBLIQUE ANGLED TRIANGLES



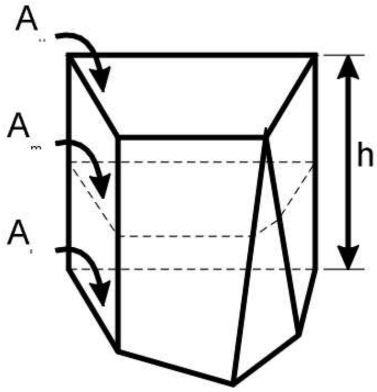
$$a^2 = b^2 + c^2 - 2\,bc \cos A$$
$$b^2 = a^2 + c^2 - 2\,ac \cos B$$
$$c^2 = a^2 + b^2 - 2\,ab \cos C$$

$$s = \frac{a+b+c}{2}$$

Known	Required					
	A	B	C	b	c	Area
a, b, c	$\cos A = \frac{b^2+c^2-a^2}{2bc}$	$\cos B = \frac{a^2+c^2-b^2}{2ac}$	$\cos C = \frac{a^2+b^2-c^2}{2ab}$			$\sqrt{s(s-a)(s-b)(s-c)}$
a, A, B			$180^0 - (A+B)$	$\frac{a \sin B}{\sin A}$	$\frac{a \sin C}{\sin A}$	
a, b, A		$\sin B = \frac{b \sin A}{a}$			$\frac{b \sin C}{\sin B}$	
a, b, C	$\tan A = \frac{a \sin C}{b - a \cos C}$				$\sqrt{a^2 + b^2 - 2\,ab \cos C}$	$\frac{ab \sin C}{2}$

Figure J-3: Trigonometric Values for Right and Oblique Triangles

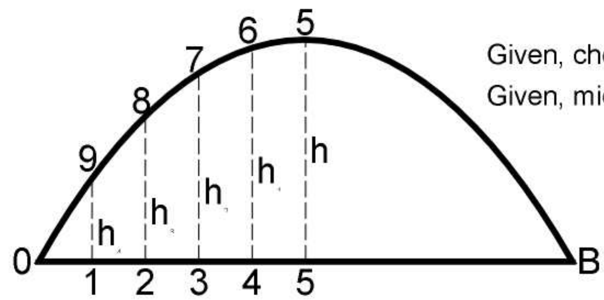
PRISMOIDAL FORMULA:



$$\text{Volume} = (A_u + A_L + 4A_m)\frac{h}{6}$$

1. Prismoid has parallel bases.
2. Middle area is not ½ sum of upper and lower base areas.
3. Average end areas give slightly larger answers.
4. To use formula, upper and lower bases will be cross sections.

PARABOLA



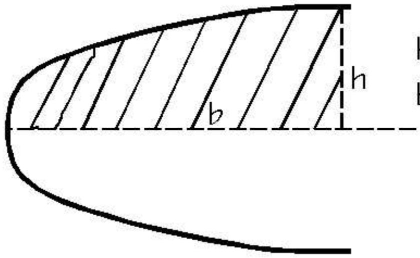
Given, chord = OB

Given, middle ordinate = h

To draw a parabola to scale, number the equally spaced ordinates between the end and center as shown; then, $h_1 = (h)\frac{9x1}{5x5}$, $h_2 = (h)\frac{8x2}{5x5}$, etc.

Figure J-4: Prismoidal Volume and Parabolic Height Calculations

PARABOLA



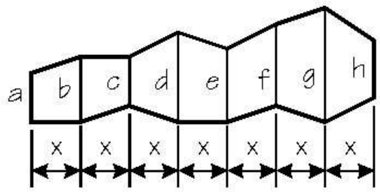
h = height
b = base

Area of Shaded Portion:

$$A = \frac{2bh}{3}$$

(Approximately)

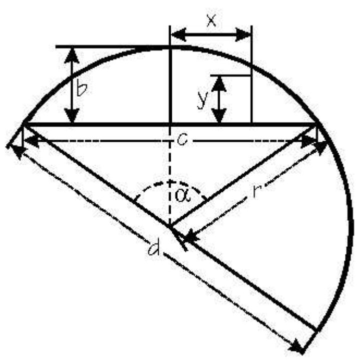
IRREGULAR FIGURE



x = x

$$A = \left(\frac{a}{2} + b + c + d + e + f + g + \frac{h}{2}\right)x$$

PROPERTIES OF THE CIRCLE



Circumference of Circle of Diameter 1 = π = 3.14159265

Circumference of Circle = $2 \pi r$

Diameter of Circle = Circumference X 0.31831

Diameter of Circle of equal periphery as Square = side X 1.27324

Side of Square of equal periphery as Circle = diameter X 0.78540

Diameter of Circle circumscribed about Square = side X 1.41421

Side of Square inscribed in Circle = diameter X 0.70711

Length of Arc = a

Arc, $a = \frac{\pi r \alpha}{180^\circ} = 0.017453\, r \alpha$ **Angle,** $\alpha = \frac{180^\circ a}{\pi r} = 57.29578 \frac{a}{r}$

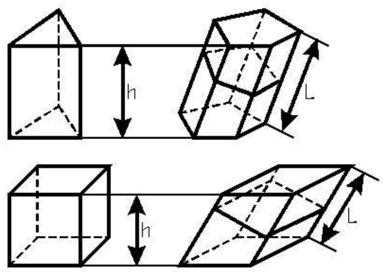
Radius, $r = \frac{4b^2 + c^2}{8b}$ **Diameter,** $d = \frac{4b^2 + c^2}{4b}$ **Chord,** $c = 2\sqrt{2br - b^2} = 2r \sin \frac{\alpha}{r}$

Rise, $b = r - \frac{1}{2}\sqrt{4r^2 - c^2} = \frac{c}{2} \tan \frac{\alpha}{4} = 2r \sin^2 \frac{\alpha}{4}$

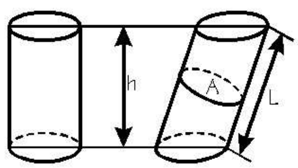
Rise, $b = r + y - \sqrt{r^2 - x^2}$ $y = b - r + \sqrt{r^2 - x^2}$ $x = \sqrt{r^2 - (r + y - b)^2}$

Figure J-5: Areas of Irregular Figures and Geometric Properties of a Circle

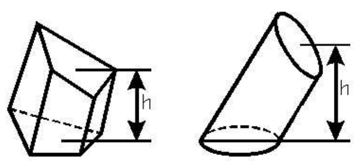
SURFACE AND VOLUME OF SOLIDS



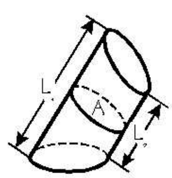
	GIVEN	SOUGHT
PRISM (RIGHT OR OBLIQUE, REGULAR OR IRREGULAR, PARALLELOPIPED)		
	Perimeter. P, perpendicular to sides; lateral length, L.	Lateral Surface = PL
	Area of base, B; perpendicular height, h.	Volume = Bh
	Area section perpendicular to sides, A; lateral length, L.	Volume = AL



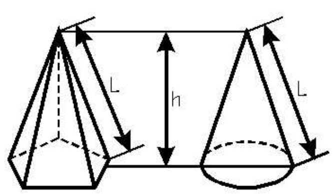
	GIVEN	SOUGHT
CYLINDER (RIGHT OR OBLIQUE, CIRCULAR OR ELLIPTIC)		
	Perimeter of base, P_b; perpendicular height, h.	Lateral Surface = $P_b h$
	Perimeter, P, perpendicular to sides; lateral length, L.	Lateral Surface = PL
	Area of base, B; perpendicular height, h.	Volume = Bh
	Area of section perpendicular to sides, A; lateral length, L.	Volume = AL



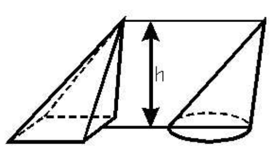
	GIVEN	SOUGHT
FRUSTRUM OF ANY PRISM OR CYLINDER		
	Area of base, B; perpendicular distance from base to center of gravity of opposite face, h.	Volume = Bh



	GIVEN	SOUGHT
FRUSTRUM OF CYLINDER		
	Area of section perpendicular to sides, A; maximum lateral length, L_1, and minimum, L_2.	Volume = ½ $A(L_1+L_2)$



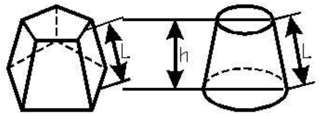
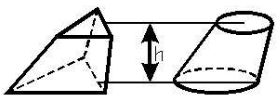
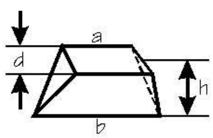
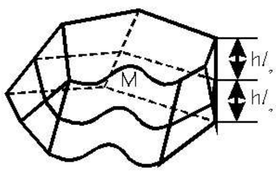
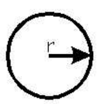
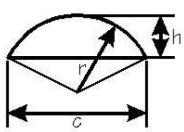
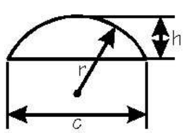
	GIVEN	SOUGHT
PYRAMID OR CONE (RIGHT AND REGULAR)		
	Perimeter of base, P_b; slant height, L.	Lateral Surface = ½ $P_b L$
	Area of base, B; perpendicular	Volume = 1/3Bh



	GIVEN	SOUGHT
PYRAMID OR CONE (RIGHT OR OBLIQUE, REGULAR OR IRREGULAR)		
	Area of base, B; perpendicular height, h.	Volume = 1/3Bh = 1/3 the volume of prism or cylinder of same base and perpendicular height or ½ the volume of hemisphere of same base and perpendicular height.

Figure J-6: Surface and Volume of Solids

SURFACE AND VOLUME OF SOLIDS

	GIVEN	*SOUGHT*
	**FRUSTRUM FOR PYRAMID OR CONE** (RIGHT AND REGULAR PARALLEL ENDS) Perimeters of base, P_b, and top, P_t; slant height, L. Areas of base, B, and top, T; perpendicular height, h.	Lateral Surface = $\frac{1}{2} L (P_b + P_t)$ Volume = $1/3\, h (B + T + \sqrt{BT})$
	**FRUSTRUM FOR ANY PYRAMID OR CONE** (PARALLEL ENDS) Areas of base, B, and top, T; perpendicular height, h.	Volume = $1/3\, h (B + T + \sqrt{BT})$
	**WEDGE** (PARALLELOGRAM FACE) Length of edges, a and b; perpendicular height, h; perpendicular width, d.	Volume = $\frac{1}{6} dh (2a + b)$
	**PRISMATOID** Areas of base, B, top, T, and of a section, M, parallel to and midway between base and top; perpendicular height, h.*	Volume = $\frac{1}{6} h (B + T + 4M)$
	**SPHERE** Radius, r.	Area = $4\pi r^2$ Volume = $\frac{4}{3} \pi r^3$
	**SPHERICAL SECTOR** Radius, r; length of chord, c; height, h.	Area = $\frac{\pi r}{2} (4h + c)$ Volume = $\frac{2}{3} \pi r^2 h$
	**SPHERICAL SEGMENT** Radius, r; length of chord, c; height, h.	Curved Surface = $2\pi rh = \frac{\pi}{4} (4h^2 + c^2)$ Volume = $\frac{\pi}{3} h^2 (3r - h) = \frac{\pi}{24} h (3c^2 + 4h^2)$

* This formula also applies to any of the foregoing solids with parallel bases, to pyramids, cones, spherical sections and to many solids with irregular surfaces.

Figure J-7: Surface and Volume of Solids

SURFACE AND VOLUME OF SOLIDS

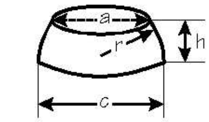
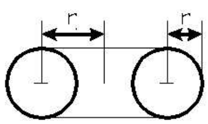
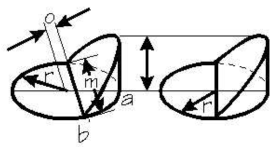
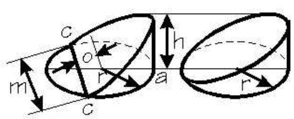
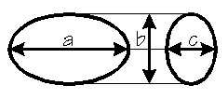
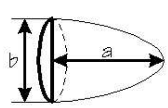
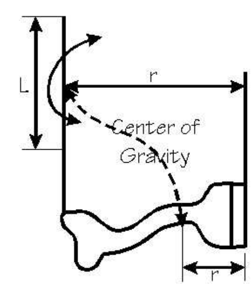
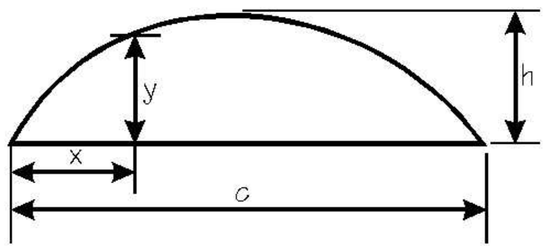
	GIVEN	*SOUGHT*
	**SPHERICAL ZONE** Radius, r; height, h; diameters, a and c.	Curved Surface = $2\pi rh$ Volume = $\frac{\pi h}{24}(3a^2 + 3c^2 + 4h^2)$
	**RING OF CIRCULAR CROSS SECTION, TORUS** Radius of ring, r, and of cross section, r_1.	Area = $4\pi 2r r_1$ Volume = $2\pi^2 r^2 r_1$
	**UNGULA OF RIGHT, REGULAR CYLINDER** Base = segment, bab' (smaller than semi-circle). Radius of cylinder, r; height, h; length of chord, m; distance from chord to axis of cylinder, o.	Convex Surface = $(rm - o \bullet \text{arc } bab')\frac{h}{r-o}$ Volume = $\left(\frac{m^3}{12} - o \bullet \text{area } bab'\right)\frac{h}{r-o}$
	Base = semi-circle, Radius of cylinder, r; height, h.	Convex Surface = $2rh$ Volume = $2/a\ r^2 h$
	Base = segment, cac' (larger than semi-circle). Radius of cylinder, r; height, h; length of chord, m; distance from chord to axis of cylinder, o.	Convex Surface = $(rm + o \bullet \text{arc } cac')\frac{h}{r+o}$ Volume = $\left(\frac{m^3}{12} + o \bullet \text{arc } cac'\right)\frac{h}{r+o}$
	Base = circle, Radius of cylinder, r; height, h.	Convex Surface = $r\pi h$ Volume = $\frac{1}{2} r^2 \pi h$
	**ELLIPSOID** Axes, a, b and c.	Volume = $\frac{1}{6}\pi abc$
	**PARABOLOID** Axes, a, b and c. The ratio of corresponding volumes of cone, paraboloid, sphere and cylinder of equal height is $\frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1$	Volume = $\frac{1}{8}\pi ab^2$
	**SURFACE AND SOLIDS OF REVOLUTION** When a plane curve or straight line revolves around an axis of revolution in the same plane, a surface of revolution is produced. A solid of revolution results from the rotation of an area around an axis in its plane. Length of curve or straight line, L; normal distance from center of gravity to axis, r; angle of revolution, α°; area of revolving plane, A.	Length of arc described by center of gravity = $2\pi r \frac{\alpha}{360}$ Surface of revolution = $2\pi rL \frac{\alpha}{360}$ Volume of solid = $2\pi rA \frac{\alpha}{360}$

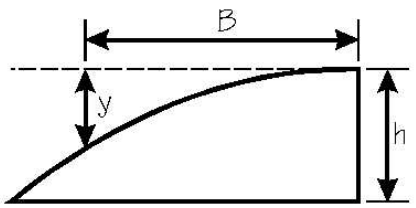
Figure J-8: Surface and Volume of Solids

PARABOLA



$$y = \frac{4hx}{c^2}(c - x)$$

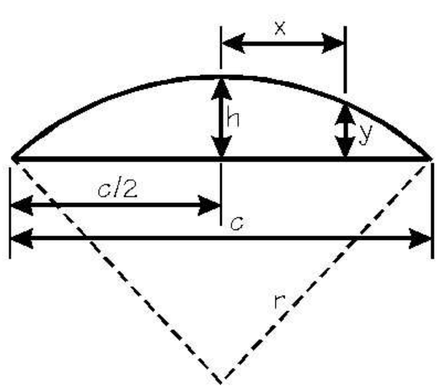
PARABOLIC CROWN ROADWAY



w = width between curbs
h = height of crown
B = distance to any point
y = distance below crown

$$y = \frac{4hB^2}{w^2}$$

CIRCULAR CROWN ROADWAY



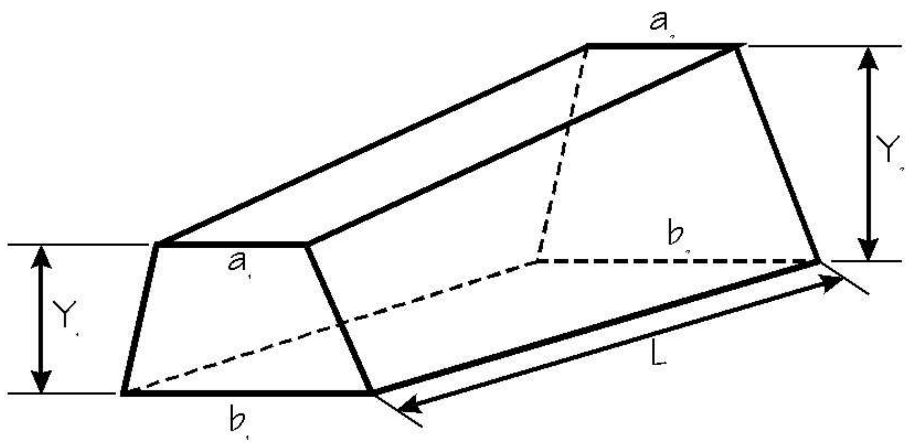
$$r = \frac{4h^2 + c^2}{8h}$$

$$y = h - r + \sqrt{r^2 - x^2}$$

$$y = h + \sqrt{r^2 - x^2} - r$$

Figure J-9: Circular and Parabolic Roadway Crown Calculations

RETAINING WALL VOLUME FORMULA



$$Volume = \frac{L\,P}{24}$$

L = Distance Between 2 Parallel Faces

$$P = h_1 V_1 + h_2 V_2 + h_t V_t$$

$$h_1 = a_1 + b_1$$

$$V_1 = 2\ Y_1$$

$$h_2 = a_2 + b_2$$

$$V_2 = 2\ Y_2$$

$$h_t = h_1 + h_2$$

$$V_t = V_1 + V_2$$